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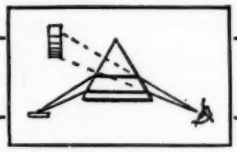

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1934

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Journal of the
OPTICAL SOCIETY *of* AMERICA

VOLUME 24 MAY, 1934 NUMBER 5

Anomalous Dispersion by Diffraction

A. H. PFUND, *The Johns Hopkins University*
(Received April 13, 1934)

Diffraction bands with black minima are produced by means of a single slit ruled in a light-absorbing film. Since these bands are at their best in the region of strong absorption, it is possible to follow with ease the anomalous dispersion of an aniline dye through the absorption band. Application of this procedure to the study of dispersion in gold films is also made.

IN[1] the construction of an improvised Young's interference apparatus two slits about a millimeter apart were ruled in an old photographic negative with a pen-knife. When the vertical interference fringes were allowed to fall on the horizontal slit of a pocket-spectroscope interference fringes of the expected type, shown in Fig. 1, were observed. However, upon narrowing the slit of the spectroscope, the unusual type of fringes, shown in Fig. 2, became apparent. Further experiments with slits ruled in various types of films showed that the essential conditions for the beaded appearance of Fig. 2 are that the film must be (1) gray and (2) that the thickness be sufficiently great to enable the film to act as a retarding medium. A single slit in gray gelatine yielded only the diffraction bands, as shown in Fig. 3. The beaded appearance of Fig. 2 is then due to the intersection of interference fringes and diffraction bands. The significant observation was made that, if a piece of black paper with a straight edge be gradually advanced toward one edge of the slit, the bands on that side of the slit disappeared when the paper had cut off all of the aperture to this one side. No change in the appearance of the fringes on the other side was observed. By rotating the spectroscope through a right angle so that its slit intercepted the central row of beads, the channelled spectrum presented in Fig. 4 was observed.[2]

The writer is indebted to Professor R. W. Wood for the suggestion that explanation of these diffraction bands was to be found in his treatment of the diffraction past a straight edge (*Physical Optics*, page 221, Third Edition (1934)). This treatment modified to meet the present

[1] After this paper was completed, Professor R. W. Wood called the writer's attention to the earlier work of G. Quincke on "lamellar diffraction" (Verdet's *Optik Deutsche Bearbeitung* 1881, Vol. 1, 292; also G. Quincke, Pogg. Annalen **132**, 321 (1867). While Quincke unquestionably gave the correct explanation of the type of diffraction discussed in the present work, he used neither absorbing lamellae or spectroscope and hence did not describe the most interesting phenomena observed in connection with this investigation.

[2] Simple calculation showed that this channelled spectrum obeyed the relation $(\mu-1)t=n\lambda_1\lambda_2/(\lambda_2-\lambda_1)$ where μ=refractive index of gelatine, t=thickness of gelatine, and n=number of fringes lying between wave-lengths λ_1 and λ_2. The calculated thickness, assuming $\mu=1.5$, was 0.029 mm, while that actually measured was 0.030 mm.

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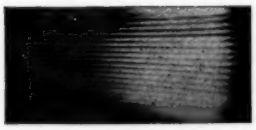

FIG. 1. Interference fringes. 2 slits in gray gelatine wide spectroscope slit.

FIG. 2. Same as Fig. 1. Narrow spectroscope slit.

FIG. 3. Diffraction by single slit in gray gelatine.

FIG. 4. Channelled spectrum.

conditions, leads to the construction shown in Fig. 5. Here, S is a narrow slit cut in absorbing but non-retarding medium. The coaxial cylindrical wave fronts A, B, C originating at S (and drawn one wave-length apart) cut the plane wave fronts L, M, transmitted by the absorbing film. The diffraction maxima at these points of intersection form bands whose separations are W_1, W_2, W_3. Leaving out of consideration for the present, the question of phase differences[3] between the cylindrical and plane waves, it is obvious that the separation of the bands to one side of the center is the same as that shown by diffraction past a straight edge. Bands of this type are, however, more or less washed out whereas, the bands produced by a slit cut in an absorbing film are able to reveal black minima since the intensity of the plane waves may be reduced by absorption in the film to the point at which destructive interference with the cylindrical waves is complete. It will be shown later that, the greater the absorption in the film, the narrower must the slit be in order that black minima be obtained.

If the absorbing film be covered by a retarding medium, as indicated by the dotted lines in Fig. 5, the plane wave front M will be retarded and will lie at M'. As a result, the bands of separation W_1 move out from the center, their new separation being W_1'. In general then an increase in the retardation in the absorbing film causes the bands to move out while decrease in the retardation causes the bands to move in toward the center.

Again neglecting phase changes at the edges of the slit, it is evident that the retardation between rays 1 and 2 in Fig. 5 is $\Delta=(\mu-1)t$. Since the retardation, expressed in terms of wave-lengths increases as μ decreases it is evident that any fringe of order n must move away from the center as λ is decreased. This accounts for the flare of the fringes toward the violet in Fig. 4.

APPARATUS

The arrangement used to demonstrate Young's interference fringes or diffraction pattern of the Fresnel type is too well known to require description. A right-angle carbon arc was used as a source of light. The distance from the first slit to the plane of the diffracting slits was about 60 cm and the distance from the latter to the slit of the spectroscope varied, on occasion, from 20 to

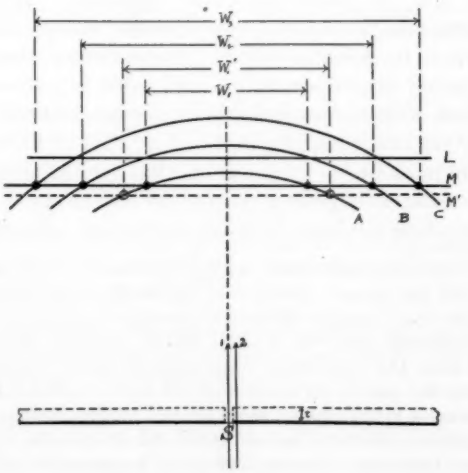

FIG. 5.

[3] A. Sommerfeld, Math. Annalen **47**, 317 (1895). Drude, *Lehrbuch der Optik* 188 (1900).

45 cm.[4] The spectroscope was of the Bausch and Lomb "pocket" type. It was mounted in one end of a cylindrical cardboard tube 20 cm long and 8 cm in diameter the slit of the spectroscope being perpendicular to the interference fringes. An improvised plateholder was inserted at the opposite end of the tube. By adjusting the position of the eyepiece lens, a brilliant spectrum 2 cm high and 5 cm long (4300–6600A) was formed. When the slit was narrowed, the two yellow mercury lines could be resolved with ease. Since the diffraction bands to be studied were rather close together, the magnification of 3.5 realized in the construction of this "spectrograph" imparted a very acceptable separation to the fringes. Exposures of 5 minutes on Wratten Panchromatic plates yielded vigorous negatives. The films of gold studied were prepared by cathode sputtering. Films of aniline dyes were obtained by pouring a solution of the dye in ethyl alcohol on the surface of a heated sheet of quarter inch plate glass. When drying took place within a few seconds, brilliant films were obtained. Narrow slits were ruled on the various films with fine sewing needles.

FIG. 6. Single slit in grey gelatine acetone. $\mu = 1.36$.

FIG. 7. Single slit in grey gelatine arochlor. $\mu = 1.62$.

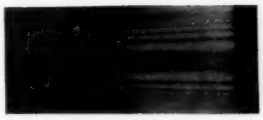

FIG. 8. Diffraction pattern due to narrow wire.

EXPERIMENTAL

To study more closely the matter of the flare of the bands shown in Fig. 3, a single slit was cut in a gray film of gelatine, this was then overlaid with a plate of clear glass and some acetone was flowed into the space between the plates. Since the refractive index for acetone is 1.36 as against 1.5 for gelatine, it is clear that the plane waves passing through gelatine and acetone are retarded relative to the cylindrical waves travelling through acetone only. In consequence the fringes flare toward the violet, as in Fig. 6. If now the acetone be replaced by arochlor of refractive index 1.62, the previous conditions are reversed and the fringes flare toward the red, as in Fig. 7.

Experiments were next carried out with single slits ruled in various types of absorbing films. For purposes of measurement it is necessary that the diffraction pattern for a single slit, ruled in an absorbing but non-retarding film, be obtained. No such film exists. Its performance may, however, be simulated by photographing the diffraction pattern produced by a narrow wire. Such a photograph for a wire 0.07 mm in diameter, presented in Fig. 8, shows that the fringes become more widely separated as one passes from the blue to the red. In striking contrast to this is the photograph, Fig. 9, obtained from a single slit ruled in a dry film of aniline violet. From the narrowing of the fringes in the green it is evident that here the refractive index has decreased—in the red, however, where the fringes have widened, the refractive index has increased. In the blue, where the dye-film is very transparent the fringes are poorly defined, while in the green where the absorption band lies and where anomalous dispersion sets in, the fringes

FIG. 9. Single slit in aniline violet. Anomalous dispersion.

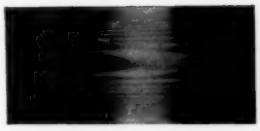

FIG. 10. Single slit in aniline green. Anomalous dispersion.

[4] Unfortunately, no record was kept of this latter distance, hence an inter-comparison of the various fringe-widths, in general, is precluded. In the following groups: (3, 4) (6, 7) (8, 10, 12, 13) the distance, while varying from group to group, remained constant within any given group.

are at their best. This procedure, then, is unique in that it yields the best results within the absorption band where the usual methods, involving the use of acute-angled prisms[5] or thin films studied by means of the Michelson interferometer,[6] encounter their greatest difficulties. A similar spectrogram for aniline green, which has an absorption band in the red, is given in Fig. 10.

The important phenomena associated with anomalous dispersion are presented in Fig. 11 where the phenomenon of absorption (of aniline violet in water) is shown in *A*, the anomalous dispersion in *B*, and the reflection (with its minimum on the blue side of the absorption band where the refractive index of the dye approaches that of air) at *C*.

In Fig. 12 is presented the pattern obtained with a very narrow slit ruled in a heavy film of gold which appeared green in transmitted light. Since this film transmitted but 2 percent of the incident light the slit was made but 0.03 mm wide. In comparison with the pattern obtained on the same scale for a narrow wire (Fig. 8) it is evident that the narrowing of the fringes in Fig. 12 indicates a refractive index for gold which is less than unity in the red. In Fig. 13 is shown the pattern for a colloidal film gold which appeared pink in transmitted light. The constriction of the fringes in the green (where an absorption band lies) and the subsequent expansion toward the red would seem to indicate anomalous dispersion for colloidal gold also.

In the preceding it has been attempted to set forth a few qualitative applications of an extremely simple experimental procedure. Experiments of a quantitative nature are under way.

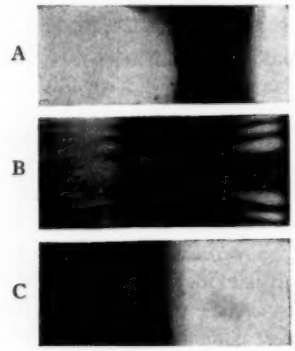

FIG. 11. Aniline violet.

FIG. 12. Single slit in green gold film.

FIG. 13. Single slit in colloidal gold. Anomalous dispersion.

FIG. 14. Helium comparison spectrum.

[5] A. Pfueger, Ann. d. Physik **65**, 113 (1898).
[6] Wood and Magnusson, Phil. Mag. **17**, 36 (1910).

Sensitivity of Photographic Films to X-Rays at Low Temperatures

G. E. M. JAUNCEY* AND H. W. RICHARDSON, *Washington University, St. Louis, Missouri*

(Received January 15, 1934)

1. INTRODUCTION

PHOTOGRAPHIC density of a developed film is defined by

$$D=\log_{10}(I/T) \tag{1}$$

where I and T are the intensities of light from the microphotometer lamp passing respectively through the unblackened and the blackened part of the film. In 1914 Friedrich and Koch[1] found that, if the total x-ray energy falling on a film in a given time is varied by means of a rotating sector, the graph of density against x-ray energy is a straight line passing through the origin. Later, Glocker and Traube[2] found that the density produced by x-rays of a given wave-length and intensity is proportional to t the time of exposure to the x-rays, and that the Schwarzschild constant is unity for x-rays, for D less than 1.0. More recently, Brindley and Spiers[3] have reported a similar relation between D and t for x-rays so long as D is less than 1.4, while Rogers[4] reports that D is proportional to t for films exposed to γ-rays so long as D is less than 0.8. In all these cases the film was at room temperature. However, for certain investigations in this laboratory it was necessary to know the relation between D and t for a film at liquid air temperature. Although Eggert and Luft[5] have published a detailed summary of the effect of temperature on the sensitivity of films to light, x-rays and alpha-rays, yet their temperatures did not extend below 213°K. We have therefore obtained curves of D against t for films at 103°K ("liquid air temperature" as measured in our apparatus by a thermocouple in contact with the film), at 203°K (dry ice in butyl alcohol) and at 295°K (room temperature). A more recent paper by Eggert[6] is also of interest.

2. EXPERIMENTAL RESULTS

X-rays from a tungsten target tube operated at 55 kv peak and 6 m.a. passed through 1.065 cm of aluminum to the film which was 12 ft. from the tube. The effective wave-length was 0.33A. Strips cut from Eastman Ultra Speed Duplitized x-ray films were used. The photographic densities were measured by means of the microphotometer whose use has already been described by Claus.[7] Our results are shown in Fig. 1. The points indi-

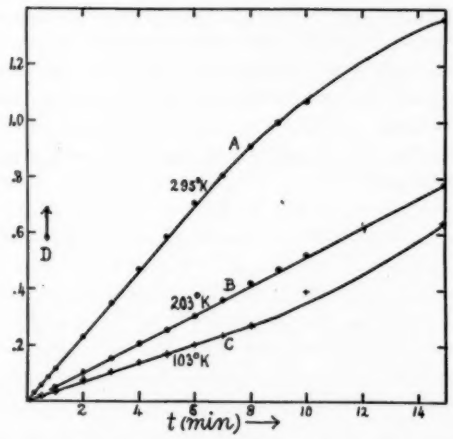

FIG. 1. Graphs of density against exposure time for different film temperatures. Films exposed to x-rays.

* The senior author was aided in part by a grant from the Rockefeller Foundation to Washington University for research in science.

[1] W. Friedrich and P. P. Koch, Ann. d. Physik **45**, 399 (1914).

[2] B. Glocker and W. Traube, Phys. Zeits. **22**, 345 (1921).

[3] G. W. Brindley and F. W. Spiers, Phil. Mag. **16**, 686 (1933).

[4] J. S. Rogers, Proc. Phys. Soc. (London) **43**, 59 (1931).

[5] J. Eggert and F. Luft, Veroeff. Wiss. Zentral-Lab. **2**, 9 (1931).

[6] J. Eggert, Zeits. f. tech. Physik **14**, 686 (1933).

[7] W. D. Claus, Phys. Rev. **38**, 604 (1931).

cate the respective average values for several films. All films were developed under the same conditions.

3. Remarks

It is seen that for each film temperature the experimental points fall upon a straight line passing through the origin so long as the density is not too great. The curve for 295°K is slightly concave to the axis of t for $D>1.0$, the curve for 203°K is straight up to $D=0.8$, while the curve for 103°K seems to be convex above $D=0.3$. However, we have obtained a point at $t=20$ min. (not shown in Fig. 1) for 103°K and this indicates that the convexity may not be as great as shown in the figure. As other observers have found, our curves show that there is no inertial period for x-rays. Confining ourselves to the straight portions of the curves, the slopes are proportional to 1, 0.452, 0.388. These numbers give the relative sensitivities at 295°, 203° and 103°K, respectively. The exposure times for equal densities are therefore proportional to 1, 2.21 and 3.07. If "H and D" curves[8] are plotted for 203° and 103°K from our results, it is found that these curves can be obtained from the curve for 295°K by horizontal shifts of this curve to the right of amounts log 2.21 and log 3.07, respectively. The "H and D" curves in Figs. 4a and 4b of Eggert and Luft's paper show the same sort of horizontal shift for different film temperatures. In agreement with Eggert and Luft we find that the change of sensitivity depends only upon the film temperature at the time of irradiation with x-rays —cooling before or after irradiation has no effect.

Dr. Claus has called our attention to Silberstein's theory,[9] which suggests that, if a silver halide grain is made developable by a single absorbed photon,

$$\log (I/T)=\alpha t, \tag{2}$$

where α is a constant. This agrees with the straight parts of our curves. However, it is hard to see why the chance of a grain being hit should depend upon its temperature. Perhaps the number of sensitive nuclei per grain is affected by the temperature.

Eggert and Luft find that the sensitivity is a linear function of the temperature between 213 and 348°K, but that this rule breaks down below 213°K. The slopes of our curves do not show this linear relation with the temperature, since our temperatures of 103 and 203°K are below 213°K. Possibly at temperatures below 213°K the sensitivity approaches a constant value as the temperature is lowered.

Combining our findings with those of Eggert and Luft, we may say that there are two linear relations for photographic films irradiated by x-rays—(1) for a given temperature, the density is proportional to the time of exposure for densities not too great, and (2) the slope of the straight portion of the density-time curve is a linear function of the temperature so long as the temperature is not too low.

[8] *See* Dobson, Griffith and Harrison, *Photographic Photometry.*

[9] L. Silberstein, Phil. Mag. **5**, 464 (1928) and **9**, 787 (1930).

Interferometer Method for Measuring the Amplitude of Vibration of Quartz Bar Crystals

SAMUEL H. CORTEZ,* *Mississippi State College, State College, Mississippi*

(Received January 15, 1934)

A simple and convenient method is described for measuring the amplitude of vibration of a quartz bar crystal excited piezoelectrically. A silvered face of the quartz bar is substituted for the movable mirror of a Michelson interferometer. White light fringes are employed in the form of a narrow band. When the crystal is made to vibrate the narrow band separates into two bands the separation of which is proportional to the amplitude.

INTRODUCTION

SEVERAL investigators have attempted, with various degrees of success, to measure the amplitude of vibrating quartz crystals. Fujimoto,[1] by varying the voltage impressed across the electrodes of a quartz resonator was able to estimate the amplitude by observing variations in the clearness of the monochromatic interference fringes formed by using the crystal as one of the mirrors of an interferometer. Osterberg[2] and Dye[3] have shown that quartz plates do not vibrate as a whole, that on the contrary, when they are excited piezoelectrically some parts of the face of the crystal execute a type of harmonic motion while other parts remain at rest. Osterberg has proposed a formula for calculating the amplitudes of vibration of the various moving parts. The data for substituting in the formula are obtained from measurements taken from photographs of the interference fringe patterns. The method of Osterberg would not apply to the measurement of the amplitude of bar crystals which vibrate as a whole. The method of Fujimoto is not readily applicable to the measurement of the amplitude of crystals vibrating as oscillators. The method described in the present article is applicable to the measurement of amplitudes of crystals or any vibrating surface which is plane and moves in a direction perpendicular to the surface and remains plane during the execution of the vibration.

DESCRIPTION OF APPARATUS

The crystal whose amplitude is to be measured is ground and polished on one face and then silvered. The silvered face is then substituted for the movable mirror of an ordinary laboratory Michelson interferometer. Reasonable care should be taken in polishing in order that the fringes be straight, as a straight narrow band of white light fringes is essential for an accurate measurement.

A brass plate one-quarter of an inch thick and one inch square to act as the lower electrode is placed on the carriage of the interferometer after the movable mirror has been removed. A thin sheet of some insulating material such as mica is usually placed between the electrode and the carriage. Creeping of the crystal while vibrating may be prevented by grooving the electrode slightly to fit the crystal. The crystal is placed on the electrode with the silvered face at right angles to the direction of motion of the carriage. The upper electrode consisting also of a plate of brass is held by means of a clamp and stand directly above the crystal and is separated from it by a small air gap. This air gap permits greater freedom for the vibration of the crystal.

The white light fringes are produced by using the silvered crystal face as the movable mirror. When the fringes are once found adjustments

* Based on work done at Indiana University.

[1] Tadashi, Fujimoto, Doctor's Thesis, Ohio State University, 1927.

[2] H. Osterberg, J. Opt. Soc. Am. **22**, 19 (1932).

[3] W. R. Dye, Proc. Roy. Soc. **A138**, 1 (1932).

can be made so that they form a narrow brilliantly colored band with the characteristic central black fringe. These adjustments are made by combining a slight tilting of the stationary mirror of the Michelson interferometer about a horizontal axis with a slow rotation about a vertical axis. By controlling the amount of these two adjustments the band of white light fringes can be made to appear either horizontal or vertical and to have any desired width. A short focus telescope with a scale mounted in the eyepiece should be used to observe the fringes.

The crystal is made to vibrate and the narrow colored band is seen to separate into two bands of faint yet easily visible brown fringes. These two bands are produced by the interference of the reflected light from the face of the crystal at the two extreme positions of its vibratory path where for an instant the crystal is stationary. The distance between the two positions where the velocity is zero is equal to twice the amplitude of the crystal. This distance is proportional to the separation of the interference bands. To find the actual amplitude in centimeters it is necessary only to observe the number of telescope scale divisions by which the bands are separated. The distance the crystal must be moved when not vibrating to displace the central black fringe one telescope scale division is easily measured with the calibrated screw of the interferometer. By having this distance, the length of the path of vibration can be readily calculated.

It is not necessary to have an expression for the brightness of the fringes produced when the crystal is vibrating. Only the separation of the fringes is observed and the separation is dependent only on the extreme positions of the face of the crystal in its vibratory path. Osterberg[4] has developed an expression involving the fringe brightness of monochromatic fringes as a function of the amplitude for the case of a Michelson interferometer in which one of the returning mirrors executes simple harmonic motion in a direction perpendicular to its plane. The expression applies only for monochromatic fringes and for comparatively small amplitudes. In the present case the brightness of the white light fringes at each extreme position might be expressed by a formula similar to Osterberg's but since it is the separation and not the brightness of the bands which is observed it was thought best to leave the derivation of such an expression as the basis of a future investigation. The central fringe is easily located as the band is narrow and consists only of six or seven fringes.

[4] H. Osterberg, J. Opt. Soc. Am. **22**, 20 (1932).

Results

The crystal used in the experimental work was a Curie cut element whose dimensions are: 1 cm in the *e* direction, 2.6 cm in the *o* direction, and 7.22 cm in the *t* direction, where *e* is the direction of the electric axis parallel to one of the faces, *o*, the direction of the optic axis, and *t* a direction at right angles to the plane containing the *e* and *o* axes. The natural frequency of the crystal is 37.4 kilocycles. Several measurements of the amplitude were made for the crystal vibrating under various conditions. The amplitude in the present case was found to be proportional to the radiofrequency current in the tank circuit. The readings are given therefore for various values of the deflections of the radiofrequency galvanometer which was connected in series with the variable condenser in the plate circuit. The radiofrequency plate current was varied by changing the setting of the variable condenser in the plate circuit which is permissible within certain limits when the crystal is being made to vibrate in an oscillating circuit.

In taking the data it was found necessary to measure the displacement of the crystal which would produce a fringe shift of fifty telescope scale divisions before and after each trial to make sure that the crystal had not shifted, as any change in the fringe width due to a slight twisting of the crystal would produce a corresponding change in the displacement. These measurements are shown in the table for each trial. Tables I–III show the results for four trials.

Table I gives the positions of the two bands on the telescope scale for various deflections of the radiofrequency galvanometer in the plate circuit.

Table II shows the actual separation of the bands in telescope scale divisions obtained by subtracting the scale positions of the two bands as given in Table I.

TABLE I. *Positions of the two bands on the telescope scale.*

Trial	Dial readings for fringe shift of 50 divisions	Plate circuit radiofrequency galvanometer readings. 10	20	30	40	50
1	75.7–72.8	18–31	15–34	13–36	12–38	
2	75.7–72.8	19–31	15–34	13–36	12–38	11–40
3	27.7–24.7	19.5–31	15–33	11–33	6–31	2–31
4	27.4–24.5	19–32	17–34	15–37	13.5–39	10–40

TABLE II. *Actual separation of bands in telescope scale divisions.*

Trial	Plate circuit radiofrequency galvanometer reading. 10	20	30	40	50
1	13	19	23	26	
2	12	19	23	26	30
3	12.5	18	22	25	29
4	13	17	22	25.5	30

TABLE III. *Calculated amplitudes in centimeters.*

Trial	Plate circuit radiofrequency galvanometer reading. 10	20	30	40	50
1	0.0019	0.0028	0.0033	0.0038	
2	0.0017	0.0028	0.0033	0.0038	0.0044
3	0.0019	0.0027	0.0033	0.0038	0.0044
4	0.0019	0.0025	0.0032	0.0037	0.0044

Table III shows the calculated amplitudes in centimeters. The following sample calculation shows how these results were obtained.

$$\text{Amplitude} = \frac{2.9 \times 13}{200 \times 10 \times 50 \times 2} = 0.0019 \text{ cm.}$$

The 2.9 represents the divisions on the dial of the interferometer screw through which it had to be turned to cause the central black fringe to shift fifty telescope scale divisions. Each division on the dial represented one-two hundredth of a millimeter. The factor ten converts the answer to centimeters and the division by two converts the total path into amplitude.

The precision of the measurements is determined by the accuracy with which the separation of the bands can be read on the scale of the telescope eyepiece and with which the displacement of the movable mirror of a Michelson interferometer can be read. The author estimates that the limit of the precision of the eyepiece scale readings is of the order of two percent while it is about three percent for the interferometer screw. It then appears that the limit of the precision of the results given is of the order of five percent.

CONCLUSIONS

It is seen from the tables that there is fairly good agreement between the results of the several trials. The greatest variation occurred at the smaller amplitudes when the impedance of the plate circuit had just reached the proper value to react back through the plate-grid capacitance and compensate the grid circuit losses thus giving rise only to oscillations of a small amplitude, a fact which might account for some of the variation.

The method might find application in the measurement of amplitudes of vibration of any object where the amplitudes are small.

The writer wishes to acknowledge his gratitude as well as his indebtedness to Dr. Arthur L. Foley of Indiana University under whose direction the work was done.

Area and Brightness of Stimulus Related to the Pupillary Light Reflex

M. LUCKIESH AND FRANK K. MOSS, *Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland*

(Received April 12, 1934)

RESEARCHES pertaining to pupillary phenomena may often have a dual rôle in visual science. They not only indicate the influence of the pupil in controlling retinal illumination, but also may be a practical means for revealing the effects of complex psychophysiological factors which otherwise might defy appraisal. For example, it has been shown (1) that elderly people require a higher intensity of illumination than young people do as a result of the gradual contraction of the pupil with age;[1] and (2) that there is a definite dilation of the pupil following several hours of exacting visual work,[2] which may be interpreted, at least relatively, as an indication of bodily fatigue. The transient pupillary phenomena, as well as the permanent changes, are often extremely important in seeing as a universal human activity.

A primary objective of the present research was to investigate the usefulness of pupillary phenomena as criteria for appraising the effects of light-sources seen fortuitously, or by necessity, as the eyes fixate on various elements of the visual field. As both the area and the brightness of light-sources are involved, a determination of the influence of these factors on the pupillary light reflex becomes a phase of the problem. Some data pertaining to this subject have been presented recently by Ferree, Rand and Harris.[3] In general, their results indicate that the size of the pupil is a function of both the area and the brightness of the stimulus field. These factors were varied independently over a range in brightness from approximately 1 to 20 millilamberts and over a range in size of field from 20 to 120 degrees. These ranges are of such an order that the data are not directly applicable to certain practical phases of the present problem. However, the data have considerable theoretical bearing on the general subject.

It has been stated by Hess,[4] as a conclusion from a rather quantitative research, that the pupillo-motor nerves are confined largely to a small central area of the retina. A casual comparison of the results of the researches of Hess and of Ferree, Rand and Harris might lead to the assumption that they are in contradiction, since the latter data indicate that the size of the field affects the size of the pupil over a large portion of the retina. It will be shown later in this discussion that the experimental results obtained in these two researches are not necessarily in disagreement. The interpretation of these results into practice is difficult due to the restricted ranges in the variables, in the one case; and because of a relatively insensitive method of detecting pupillo-motor effects, in the other.

Specifically, the present research involves a determination of the changes in pupillary area when the brightness and area of the stimulus are varied in such a manner as to produce a constant intensity of illumination at the eyes. A consideration of the applicability of the experimental results to lighting practice dictated, to a large extent, the magnitude of the various factors involved.

PROCEDURE

The subjects were required to fixate upon a small opaque test-object located at the center of a circular luminous area which was seen amid comparatively dark surroundings. The adjustments of apparatus necessary for the measure-

[1] Nitsche and Gunther, *On the Variation of the Diameter of the Pupil of the Eye with Age*, Mittleilungen, p. 117, December, 1930.

[2] M. Luckiesh and Frank K. Moss, *Size of Pupil as a Possible Index to Ocular Fatigue*, Am. J. Ophth. **16**, 393 (1933).

[3] C. E. Ferree, G. Rand and E. T. Harris, *Intensity of Light and Area of Illuminated Field as Interacting Factors in Size of Pupil*, J. Ex. Psychol. **16**, 408 (1933).

[4] C. Hess, *The Extent of the Pupillo-motor Areas of the Retina and Pupillo-motor Receiving Organs*, Arch. Ophth. **38**, 131 (1909).

ment of pupil size were then made and the subjects directed to fixate upon a small light-source of low brightness situated about 10 degrees from the luminous area. The time required to complete this cycle of events, as well as the time interval between cycles, was that preferred by the individual subjects. A series of five such cycles were taken for each of the six different conditions of brightness and area of the stimulus. A definite adaptation period followed each change in the brightness-level to which the eyes were exposed.

The circular illuminated fields were located at a distance of two meters from the subject. They consisted of sheets of opal glass illuminated to a uniform brightness except in the case of the largest field. The latter was a wall of the laboratory which was painted a flat white and fairly uniformly illuminated. For obvious reasons, the area of this field is not subject to exact specification. The reciprocity relationship between brightness and area of the test-fields was such that an intensity of illumination of 10 footcandles was produced at the eyes of the subject in each of the six experimental situations. This level of illumination was selected as representing a condition favorable for a practical interpretation of the results. Ocular discomfort resulting from high brightnesses imposed a practical limit to the range over which the factor of brightness was varied.

Pupillary measurements were made with a modified form of the Broca pupillometer. This instrument has been described in detail elsewhere.[5] With this device, the diameter of the pupil is measured by the distance between two pin-points of light so located in front of the right eye and so separated that the images of the pupil cast upon the retina appear tangential. Due to the high brightnesses involved in several of the stimuli, it was not feasible to use a pupillometer of a type which reflects the small pencils of light into the eye by means of a clear plate glass reflector, thus permitting the test-field to be viewed binocularly. In this research, the pupillometer proper was placed directly in front of the right eye, thereby obscuring the visual field of that eye. Since light entering the unobscured eye adds its pupillomotor influence to the other, the results obtained with the "monocular" type of instrument appear satisfactory as relative criteria for the present purpose.

A comparison of the results obtained with instruments of both types, used under identical conditions, is shown in Fig. 1. It will be noted that similar relationships between pupil size and fixational distance are indicated for both monocular and binocular vision. These data also indicate that the size of the pupil is not altered significantly even by large increases in fixational distance when the latter is 60 centimeters or greater.

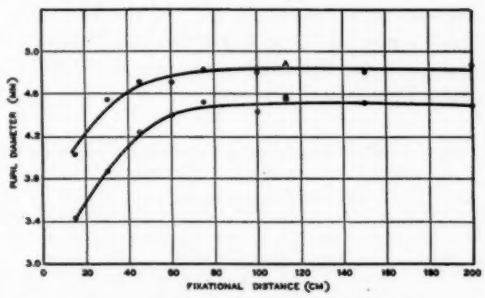

FIG. 1. The relationship between pupil size and fixational distance for a brightness-level of 0.1 millilambert. Each circle represents 129 observations involving four subjects. The results shown by curves *A* and *B* are for monocular and binocular observation, respectively. In each case, the diameter of the right eye was measured. The data shown by curve *A* were obtained with the visual field of the right eye obscured by the pupillometer.

The experimental procedure involved an adaptation period of 10 minutes preceding the initial measurements. This period was not only for the purpose of attaining a fairly constant degree of retinal adaptation,[6] but also to enable the subjects to attain a reasonable state of physical composure. During this time, the eyes were adapted to the lowest brightness and correspondingly largest area of the series of stimuli. The various stimuli were then presented to the subjects in the order of their increasing brightness. This procedure was adapted after a preliminary series of measurements established

[5] Frank K. Moss, *A Modified Broca Pupillometer*, J. Opt. Soc. Am. **22**, 735 (1932).

[6] Selig Hecht, *The Dark Adaptation of the Human Eye*, J. Gen. Physiol. **2**, 499 (1920).

the fact that the method produced essentially the same results as were obtained with alternate series from low to high brightnesses and *vice versa*. The method followed possesses some advantage in that the disturbing influence of after-images is minimized. An adaptation period of two minutes was allowed between the presentation of the different stimuli. Such a period is adequate for pupillary reflex phenomena[7] and does not unduly prolong the discomfort of looking at the bright light-sources.

Results

A series of five measurements of the diameter of the pupil were taken for each stimulus condition for each of eight subjects on each of three different days. Therefore, each of the 48 average values given in Table I represents 120 separate measurements. These data are presented graphically in Fig. 2.

Table I. *The diameter of the pupil for each of eight subjects of normal or near-normal vision as determined from 3 series of 5 measurements each.*

	Diameter of pupil in millimeters					
	Area of test-field (square meters)					
	12*	0.52	0.29	0.16	0.051	0.010
	Brightness of test-field (lamberts)					
Subject	0.036	0.27	0.46	0.82	2.50	12.9
1	3.18	2.69	2.62	2.69	2.66	2.70
2	3.15	2.87	2.68	2.69	2.90	3.20
3	3.58	3.28	3.27	3.37	3.45	3.62
4	3.33	2.99	2.90	2.94	2.96	3.07
5	4.09	3.64	3.58	3.62	3.64	3.91
6	4.91	4.14	4.04	4.09	4.40	4.50
7	4.21	3.97	3.93	4.05	4.02	4.34
8	4.00	3.43	3.26	3.33	3.45	3.64
Mean diameter	3.81	3.38	3.28	3.35	3.44	3.64
Probable error	0.09	0.08	0.08	0.08	0.08	0.09
Area	11.40	8.97	8.45	8.81	9.29	10.40

* Estimated.

A conspicuous characteristic of the data is that a minimum in pupil size is obtained for combinations of area and brightness of the field which produce a constant intensity of illumination of 10 footcandles on a vertical plane at the eyes. This minimum occurs at a brightness of approximately 0.5 lambert and an area of about 0.3 square meter

[7] P. Reeves, *Rate of Pupillary Contraction and Dilation*, Psychol. Rev. **25**, 330 (1918).

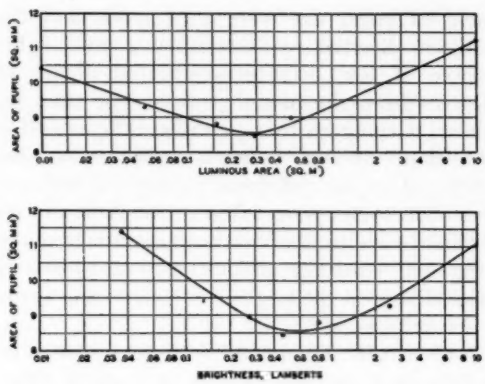

Fig. 2. The variation in pupillary area with simultaneous changes in area and brightness of a circular test-field seen amid comparatively dark surroundings by eight observers. The intensity of illumination at the eye due to the luminous test-area was always 10 footcandles.

at a distance of 2 meters, or a circle of 17 degrees visual angle. The same minimum is indicated by each of the eight subjects. It will also be noted that the relationships between pupil size and the experimental variables, as indicated by the individual subjects, are quite similar. The probable errors of the average values of pupil size are computed by using the average of each series of five measurements as a single datum. These probable error values include the systematic differences which exist between the individual subjects. Since the probable errors of the average values of pupil size are of the order of two percent, the data have quantitative significance.

Interpretations

If the population of the pupillo-motor nerves were of uniform density throughout the retina, it appears that the relationship between pupil size and the experimental variables should be represented by a straight horizontal line in Fig. 2, since a reciprocity relationship exists between brightness and area of the stimulus field. It is reasonable to conclude from the curves that the density of these reactors is not uniform and that it varies in a rather complex manner. The minimum pupil size represents the condition at which the dilation of the pupil due to a differential decrease in brightness is equivalent to the contraction due to a differential increase in

stimulus area. The data also indicate that differential changes in area are more effective in altering the size of the pupil than are the corresponding changes in brightness for the smaller stimulus areas. For the largest areas and correspondingly low brightness-levels, the reverse is true. These conclusions are supported by the data of Ferree, Rand and Harris.[3] It is evident from Fig. 2 that the region of greatest density of pupillo-motor reactors is located in an area *somewhat smaller* than that covered by the image of a stimulus area of 0.3 square meter seen at a distance of 2 meters. Assuming that the nodal point is 15 millimeters in front of the retina, this stimulus area produces a retinal image approximately 4.6 millimeters in diameter. Such an area is subtended by a visual angle of 17 degrees.

Since the macula may subtend with the nodal point an angle from 4 to 12 degrees, it seems probable that the region of greatest density of pupillo-motor reactors is located either within or quite near to the macula. This conclusion is based upon a qualitative analysis of the data. The experimental method used by Ferree[3] seems the better adapted for a quantitative survey of the density of the pupillo-motor reactors.

An analysis of the data shown in Fig. 2, in conjunction with the data of Reeves,[8] indicates that the pupillo-motor reactors are not confined entirely to a small central area, as is assumed by Hess.[4] Reeves' data indicate a relative pupillary dilation of approximately 115 percent in area for a decrease in brightness from 0.5 lambert to 0.036 lambert. The same decrease in brightness, but with corresponding increases in stimulus area (Fig. 2) produces a dilation of but 35 percent in area. This difference in pupillo-motor effect, in the two cases, is ascribed to the influence of the factor of size of the retinal image. In the first case the effect of size is constant and perhaps a maximum;* and in the second case it is a variable which eventually approaches the maximum value. Although the density of the reactors in the peripheral region may be low, the resultant pupillo-motor effect may be quite important when the area of the stimulus field is large.

Apparently the failure of Hess to detect pupillo-motor effects in retinal areas greater than perhaps 6 millimeters in diameter is due to inherent characteristics of the experimental method rather than to an absence of such reactors in the non-central region of the retina. Obviously the effectiveness of an exploring light producing a retinal image of constant size would decrease as the density of the pupillo-motor reactors decreased. Since the qualitative entopic method of observation requires rather large changes in pupil size, in order to detect such changes, it is reasonable to expect that the method would indicate a definite limit for the pupillo-motor zone. The boundary of such a zone would be indicated when the pupillary effects of the exploring light were of the same order of magnitude as the uncontrollable transient changes in pupillary size.

Lighting Practice

The application of these results to lighting problems emphasizes the empirical conditions of the experiment. However, the difficulties encountered are not unique. They are similar to those usually present in the translation from the specific conditions of a controlled research to the general and controllable conditions of practice. In the present case, the eyes were continuously exposed to the brightnesses of the test-field, except that the fixation was non-foveal between measurements. These bright areas were seen amid comparatively dark surroundings. In practice such an arrangement would represent an extreme condition.[9] However, bright areas and light-sources are frequently seen in the line of vision as the eyes fixate upon various elements of the visual field. The difference, therefore, between the laboratory and practical situation is largely a matter of degree. Hence, it may be assumed that the experimental results are applicable to practice in a relative manner. The significance of these data is augmented by the fact that the time elapsing between the beginning of the stimulus and the maximum pupillary contraction is of the order of a second.[7]

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[8] P. Reeves, *The Response of the Pupil to Various Intensities of Light*, J. Opt. Soc. Am. 4, 35 (1920).

* Reeves does not state the area of the field.

[9] M. Luckiesh and Frank K. Moss, *Seeing—A Partnership of Lighting and Vision*, Williams and Wilkins Co., Baltimore, 1931.

The results presented in Fig. 2 indicate that small areas of high brightness and large areas of low brightness, respectively, are most effective on the basis of admitting light to the retina. The pupil, under these conditions, is approximately 20 and 30 percent larger, respectively, than it is under the conditions producing the minimum size. It may be concluded that the pupillary light-reflex is the least effective as a protecting device for these extreme conditions; and that it is the most effective for stimulus areas subtending about 17 degrees (for the specified conditions) at the eye. The latter conclusion is based upon the assumption that the changes are from a lower to a higher brightness-level. The reverse is true when the change is from a higher to a lower brightness-level.

In interpreting these conclusions into lighting practice, it is to be recalled that they are based entirely upon pupillary phenomena and do not include a consideration of ocular discomfort which may result from high brightness-contrasts in the visual field. It is conceivable that the change in the size of the pupil, upon exposure to radically different brightnesses, might be used as a means for appraising the relative degree of ocular discomfort produced.[2] Such a method appears to be worth consideration in cases where the area of the light-source is constant. Obviously, when both area and brightness are variables, the pupillary reactions are too complex to be of much practical value in appraising relative ocular discomfort.

Acknowledgment is hereby made to Mr. L. L. Holladay for his assistance in certain detailed phases of the research.

The Detection of Small Color Differences in Dyed Textiles

ROBERT D. NUTTING,* *Massachusetts Institute of Technology, Department of Physics*
(Received March 15, 1934)

THE art of dyeing textiles is one of extreme antiquity and its development closely parallels that of civilization. If one inquires into the technique used by the dyer in making a color match, it is found that the methods are largely intuitive and that the dyer is guided chiefly by his many years of experience. During this time he has been able to set up an arbitrary scale of color values in terms of the concentrations of the dyestuffs with which he is familiar. The development of new color measuring instruments within the last few years has provided a tool which should prove very effective in the hands of a trained dyer and should materially assist him in meeting the ever-increasing pressure of competition.

There is little question that the trained human eye is a very sensitive mechanism for the detection of small color differences. Thus, a dyer confronted with two yellow swatches may describe one of them in a qualitative way as being redder than the other, even when the difference is very slight. He may even indicate the degree of the difference in a quantitative way by stating the difference in the amounts of the dyestuffs used. It can be seen that with this system each dyestuff or combination of dyestuffs must be considered as a special case, and the scale of color values established for one such case can not possibly be extrapolated to include mixtures of different dyestuffs. In other words, a dyer is unable to predict accurately the color that will be produced by mixing two dyestuffs if he has not already made a scale of mixture values for these materials.

The reason so many arbitrary scales of color values must be established is that each scale is made in terms of the chemical composition of the dyebath rather than in the purely physical measurements of the color itself. The eye is not an analytical instrument and is therefore unable to present its observations in any fundamental units. The process of color perception is a process of integration and, as is generally appreciated, the color of an object is determined not only by the characteristics of that object but also by the quality and quantity of the light with which it is illuminated. If this illumination is altered, the object may appear to the eye to have a different color, whereas, in truth, its fundamental characteristics have in no way been changed. It should be apparent then, that some analytical method must be employed that will give an unqualified representation of the characteristics of the object alone. The instrument for so measuring the reflectance of materials is known as a spectrophotometer, or color analyzer, and is designed to measure the amount of light reflected from the colored object at each wave-length of the visible spectrum. The spectrophotometric method not only provides a wealth of information that can be secured in no other way but it is so fundamental that the results obtained with it can be readily translated into the more familiar terms of dominant wave-length, purity and brightness, or for that matter into any other terms that may be found convenient. One is naturally prompted to ask why the spectrophotometer has not found a more general use in that most likely field, the dyeing of textiles. The answer to this is probably to be found in the fact that in the past all such measurements had to be made with a visual spectrophotometer. To obtain a complete spectrophotometric curve in this manner requires considerable time and patience. Furthermore, the data may be of questionable accuracy especially in the blue and violet portions of the spectrum because of the difficulty in making settings in this region. The development of a rapid recording photoelectric spectrophotometer at the Massachusetts Institute of Technology by Professor Arthur C. Hardy has provided a means

* Senior Fellow, The Textile Foundation.

of obtaining the necessary spectrophotometric data with adequate precision and in a matter of only a few minutes. It was in the belief that sufficient data to throw more light on the important problem of textile dyeing might be obtained with this color analyzer that this program of research, of which this paper is a part, is being sponsored by the Textile Foundation.

In spite of the general recognition of the fundamental validity of the spectrophotometric method of color specification, C. Z. Draves[1] has concluded that the human eye is a better judge of small color differences than a visual spectrophotometer. Since this entire program is based on the use of the spectrophotometric method, it seemed necessary to establish at the outset that his conclusions do not apply to photoelectric spectrophotometry with an instrument of suitable design. Draves' conclusions were based on experiments which consisted in dyeing swatches of wool with a yellow dye, and with a mixture of yellow and red dyes. The percentage of yellow dyestuff was maintained constant in every case while the amount of red was varied. Draves found that he could perceive color differences between the various swatches with his eye that did not show up in his spectrophotometric analysis. Since he confined his determinations of reflection to only one wave-length, it was felt that his conclusions needed further investigation.

The dyeings made in the experiments reported herein followed as closely as possible those described by Draves, and the spectrophotometric curves were made with the recording spectrophotometer previously mentioned. This will be described in more detail in a paper by Hardy, and only a brief summary of its operation will be given here. Light from a tungsten lamp is dispersed into a spectrum by means of a double monochromator, and a narrow spectral band is allowed to enter the photometering system. The latter consists of a set of three polarizing prisms, an integrating sphere and a photoelectric cell. One of the three prisms (a Wollaston prism) splits the monochromatic beam into two beams. These two beams are made to fall alternately upon the sample and the standard white by means of a Rochon prism rotating at constant speed of 30 revolutions per second behind the Wollaston prism. The intensities of the two flickering beams are varied by means of another Rochon prism, free to turn about its axis, in front of the Wollaston prism. Light is reflected by the sample or the standard to the whitened walls of the integrating sphere and then to the photoelectric cell, which is shielded from polarization effects by a ground glass plate. A difference between the reflectance of the sample and that of the standard causes a pulsating current in the photoelectric cell, and this is amplified sufficiently to drive a small motor. The motor rotates the Rochon prism located in front of the Wollaston about its axis until the two flickering beams reflected from the sample and the standard reach the cell with equal intensities. The magnitude of this rotation is a measure of the reflectance of the sample and may be read directly, if desired, from a divided circle. However, the rotation of this prism is also translated into a linear motion and is automatically recorded by means of a pen on a sheet of coordinate paper. The wave-length of the monochromatic light entering the photometer is altered at a constant rate and is recorded by a motion of the coordinate paper perpendicular to that of the pen. This color analyzer is a null instrument and the reflectance measurements are independent of the intensity and quality of the light source, the sensitivity of the photoelectric cell, or the gain in the amplifying circuit. In the measurements presented here the standard was always a block of magnesium carbonate freshly smoked with magnesium oxide and tested for its uniformity.

The dyeings made by Draves contained the amounts of yellow (du Pont Tartrazine O) and red (Crocein Scarlet Conc.) dyestuffs shown in Table I. Crocein Scarlet has an absorption maximum at 510 millimicrons. Draves naturally expected that the greatest effect would be produced where the absorption of this dye is a maximum and he therefore confined his spectrophotometric observations to this wave-length. He stated that the presence of "1/5000 percent of the red could be quite definitely distinguished visually because it made the yellow dyeing slightly redder in hue, whereas 1/10,000 percent of red was not always distinguishable." In other

[1] Draves, J. O. S. A. **21**, 336 (1931).

TABLE I.

Sample No.	% of Dyestuff by Weight of Cloth: du Pont Tartrazine O	% of Dyestuff by Weight of Cloth: du Pont Crocein Scarlet Conc.
1	1.0	zero
2	1.0	0.0001
3	1.0	0.0002
4	1.0	0.0050
5	1.0	0.0010
6	1.0	0.0020
7	1.0	0.0050
8	1.0	0.0100
9	1.0	0.0200
10	1.0	0.5000

words, he could always distinguish between samples 1 and 3 but not between 1 and 2. However, he could observe no differences at all between these dyeings from his spectrophotometric examinations made at 510 millimicrons.

Referring to Fig. 1, the curves of which are spectrophotometric analyses of swatches dyed in the laboratory of Dr. R. E. Rose of the E. I. du Pont de Nemours Co., Inc., and reproduced here with his kind permission, it is seen that the various dyeings (which are numbered in the order given above) are clearly differentiated. These curves are in close agreement with those consistently obtained in this laboratory on dyeings made according to Draves' technique. It should be noted that the difference in reflectance between samples 1 and 2 is approximately 3.5 percent at the red end of the spectrum. This difference is evidently significant since the color analyzer can be depended upon to reproduce the curves to within a fraction of the width of the penline. Hence, we may conclude that there is no difficulty in determining by the spectrophotometric method a color difference that is quite imperceptible to the unaided eye.

It is interesting to note that Draves described the difference between samples 1 and 3 as a difference in hue. This judgment was evidently based upon his knowledge of the effect produced by larger quantities of the red dye. Referring to curve 8 in Fig. 1, it will be seen that 0.01 percent of the red dye does definitely shift the hue toward the red end of the spectrum. The curves in Fig. 1 have been converted, according to the

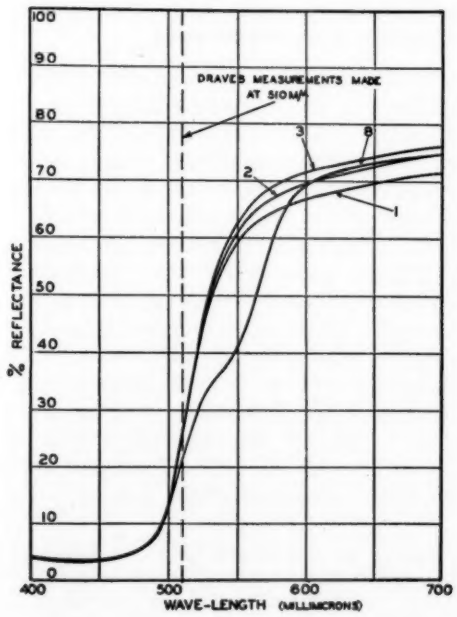

FIG. 1.

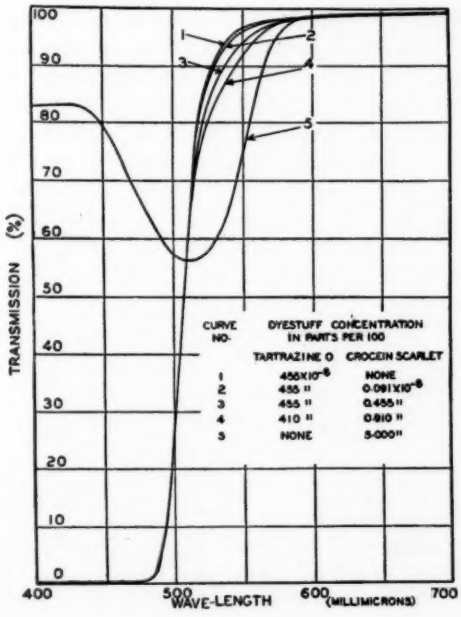

FIG. 2.

TABLE II.

Sample	Dominant wave-length (mμ)	Colorimetric purity (%)	Luminosity (%)
1	577	92	54.2
2	577	92	55.9
3	577	92	57.3
8	582	90	47.1

Standard Observer and Coordinate System for Colorimetry outlined by Judd,[2] into dominant wave-length, colorimetric purity and luminosity (brightness) with the results given in Table II. It will be seen that the principal effect of adding a small amount of Crocein Scarlet Conc. is to increase the brightness of the swatch and that larger quantities are required to produce a shift in the hue (dominant wave-length). Hence, although Draves was able to distinguish between samples 1 and 3 by merely looking at them, he was unable to correctly state the manner in which they differed.

The anomalous effect produced by small amounts of the red dye (curves 2 and 3) would undoubtedly have been detected by Draves, even with his visual spectrophotometer, had he continued his observations throughout the spectrum. This effect is a definite increase in reflectance at the red end of the spectrum when small amounts of Crocein Scarlet Conc. are added to Tartrazine O on wool. This is entirely contrary to the effect that would be expected and so far as we know has not been previously recorded. That the above curves do represent an anomalous behavior of these dyestuffs will be apparent from Fig. 2. Here it is seen that successive small additions of the red dye to a solution of the yellow dye lower the transmission in the manner that would be anticipated from Beer's law. Since these same additions actually raised the reflectance of the dyed wool, it must be concluded that this peculiar behavior is undoubtedly connected with the manner in which these dyes are taken up by the cloth. An explanation of this inconsistency will not be attempted at this time, since the purpose of this paper is merely to show the applicability of the spectrophotometric method to the detection of small color differences. It is hoped that the results given here will definitely serve to remove any prejudice that may have been formed against the precision of the spectrophotometric method and will encourage other investigators to apply it to their particular problems of color measurement.

[2] Judd, J. O. S. A. **23**, 359 (1933).

Reflecting Power of Powders in the Far Infrared Region

CARROLL W. BRYANT, *The Johns Hopkins University*

(Received January 25, 1934)

The reflecting power of quartz and marble in the crystalline form was determined for the spectral band of 80μ–150μ emitted by the Welsbach mantle. These materials were then powdered and the specular component of the radiation reflected from them was obtained as a function of the particle size and the pressure applied to them. The "Focal Isolation" method set-up for reflection measurements was employed.

INTRODUCTION

IN the past, evidence has been brought forth to show that crystals when divided into sufficiently fine particles do not reflect well in their region of metallic reflection.[1] However, if these powders were pressed, it was found that a plate so formed became a fairly good reflector in this same region of metallic reflection.[2]

It is the object of this paper to show in a quantitative manner the dependence of the reflecting power of powdered materials upon pressure.

APPARATUS

The experimental set-up used in this investigation was essentially the same as that used by Rubens and Wood in their work on the reflecting power of various substances in the far infrared region.[3] A Welsbach mantle W acts as the source of radiation. The quartz lens L_1 converges the rays upon the flat reflecting surface R of the material being studied. The rays, upon reflection from this surface, strike the small elliptical mirror r_1 and are brought to a focus at F. The radiation here passes through the filter F of opaque (to visible light) camphor soot on a nitrocellulose film and the quartz lens L_2 brings it to a focus on the thermocouple junction T of the radiomicrometer M. This filter served the purpose of keeping scattered visible light from entering the case C. It transmitted 90 percent of the energy present in this spectral region. N is a thin nitrocellulose film acting as the inner window of C. This double-walled case C surrounding the last quartz lens L_2 and the radiomicrometer M kept the temperature relatively constant. The path of the radiation is interrupted at S by a rocksalt shutter. Since rocksalt transmits the short but not the long infrared radiation, it is a suitable material for use as a shutter in the long infrared region, for, if short infrared radiation is present, it will then be a constant factor in the reflection measurements. The reading telescope was two meters from the mirror r_2. The maximum deflection when R was a silver mirror was 3.8 cm. For all measurements, a spectral band was used having wave-length limits at 80μ and 150μ[3] with a maximum at 95.8μ.[4]

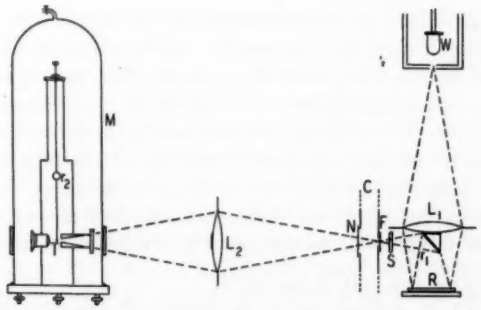

FIG. 1. Schematic diagram of apparatus.

[1] R. W. Wood, *Physical Optics* (1929), pp. 631–2; A. H. Pfund and S. Silverman, Phys. Rev. **39**, 64 (1932).

[2] R. B. Barnes, Phys. Rev. **43**, 31 (1933).

[3] H. Rubens and R. W. Wood, Phil. Mag. [6] **21**, 249 (1911).

[4] E. C. Wente, Phys. Rev. **16**, 133 (1930).

Materials

The material chosen for the main part of this investigation was marble which has the calcite crystalline form and has a strong restrahlen band at 93μ.[5] The coarser powders were separated into groups of uniform particle size by means of screens while the finer powders were separated by settling out in water.[6] Calcium carbonate particles of a smaller uniform size were obtained by precipitation from solutions of potassium carbonate and calcium chloride.

Measurement and Discussion

In all percentage measurements given, the reflection from a silver mirror was taken as 100 percent. The polished marble surface reflected 41 percent of the radiation incident upon it which is in close agreement with the value previously observed of 43.8 percent.[3]

The specular components of the radiation reflected from surfaces of the unpressed marble powders are shown on curve P_0 of Fig. 2. As the particle size becomes small compared to the wave-length, that is, less than 80 microns, there is a rapid falling off in the intensity of the scattered radiation. As the particles increase in size above 80 microns, they become equal to and then large as compared to the wave-length of the radiation directed upon them. They now assume the rôle of small mirrors and should reflect strongly. This seems to hold true when the particle size is equal to the wave-length, but beyond this point, as the particles become larger, the reflecting power goes down to less than 0.5 percent. The maximum of reflection, when the particle size is approximately equal to the wave-length can be explained by considering the size of the particles and the corresponding degree of polish assumed by a surface made up of them. When the particles are large as compared to the wave-length, a surface composed of them is so rough that only a very small percentage of the total energy directed upon this surface is specularly reflected. As the particle size is diminished,

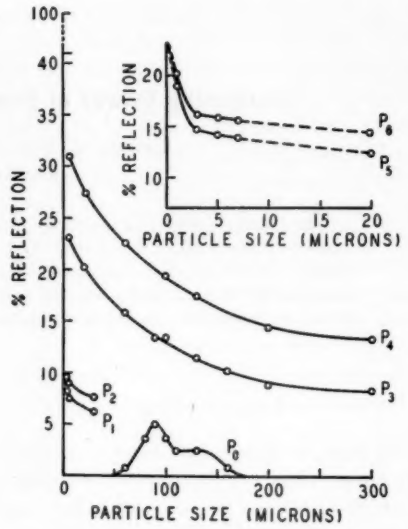

FIG. 2. The dependency of the reflecting power of plates formed from powdered materials upon the particle size and pressure. Curves P_0, P_3 and P_4—powdered marble unpressed, pressed at 10,000 and 20,000 pounds per square inch, respectively; curves P_1 and P_2—powdered quartz pressed at 10,000 and 20,000 pounds per square inch, respectively; curves P_5 and P_6—precipitated $CaCO_3$ pressed at 10,000 and 20,000 pounds per square inch, respectively.

the surface formed by these powders has a higher degree of polish with a corresponding increase in its reflecting power. However, as the particle size further decreases, the radiation is better able to penetrate into the powder surface and is finally completely absorbed when the particle size becomes less than the wave-length of the radiation used.

The reflecting powers of the marble powders pressed for three minutes at 10,000 and 20,000 pounds per square inch are shown in curves P_3 and P_4, respectively. As the particles become smaller the reflecting powers increase. Five micron particles pressed at 20,000 pounds per square inch formed a plate which had a reflecting power of about three-fourths that of the natural crystal.

The unpressed 1, 3 and 7 micron particles showed no observable reflection. The reflecting powers of these powders pressed at 10,000 and 20,000 pounds per square inch are shown in curves P_5 and P_6, respectively. Since these precipitated powders are of the same chemical

[5] Int. Crit. Tab. **5**, 261 (1929).

[6] J. H. Calbeck and H. R. Harner: *Particle Size and Distribution by Sedimentation Method*, Ind. and Eng. Chem. **19**, 58–61 (1927).

J. L. Rodda, Technical Publication No. 438 of The American Institute of Mining and Metallurgical Engineers.

composition as marble, one would expect that, when pressed at 10,000 and 20,000 pounds per square inch they would show a reflection of approximately 22.8 percent and 30.5 percent, respectively (as read off from curves P_3 and P_4) for the 7 micron particles and a higher reflection for the smaller particles. Instead, when the 7 micron particles were so pressed, they showed a 13.9 percent and a 15.5 percent reflection, which in the first case is 61 percent and in the second case 52 percent of what we would expect. This reduced reflection can perhaps be ascribed to the fact that calcium carbonate may crystallize in several forms, the ones most usually occurring being calcite and aragonite.[7] It is known that the reflecting power of aragonite is only one-third as strong as that of calcite in the region of 100μ.[8] It has been concluded that with the proper proportions of these two crystalline forms, it might be expected that the reflecting power would be reduced by from 50 to 60 percent. Points for the 5 and 20 micron particles on curves P_5 and P_6 were obtained respectively from curve P_3 by reducing the ordinates by 52 percent and from curve P_4 by reducing the ordinates by 61 percent. When so reduced, these points fall well in line with the observed experimental points for the 3 and 7 micron particles. The fact that both curves are not reduced in the same proportion indicates that increased pressure does not have the same effect on marble powders as it does upon uniformly precipitated crystals. It is interesting to note that the 1 micron particles which were made under the same conditions as the 3 and 7 micron particles showed a marked increase in reflecting power. This probably indicates that when pressure is applied to particles smaller than 3 microns recrystallization occurs. If the curves P_5 and P_6 are extrapolated further, they would probably cross at about 22.7 percent which is about 55.7 percent of the reflecting power of the marble slab, 55.7 percent being an average of the amount by which P_3 and P_4 were reduced so as to coincide with P_5 and P_6. From this it appears reasonable to conclude that if the smaller sizes of the marble powders could be obtained in large enough quantities, plates could have been formed of them which would reflect as strongly as the natural crystal.

Finally, the reflecting power of quartz, a material of very low reflecting power in this region, was studied. The unpressed powders showed no reflection. The reflecting powers of these powders pressed at 10,000 and 20,000 pounds per square inch are graphically represented in curves P_1 and P_2, respectively. One face of a large quartz prism reflected 13.1 percent of the energy. The calculated value for the reflecting power R as obtained from the formula,

$$R=100\ (n-1)^2/(n+1)^2,$$

where n is the average index of refraction of quartz for these waves of 2.165, was found to be 13.6 percent which is in favorable agreement with the value found experimentally.

Conclusions

If a high degree of reflection is to take place the effective extinction coefficient must be such that the amplitude of the incident wave is reduced almost to zero within a thin layer of the material approximately one wave-length thick. Application of pressure to any material compresses the particles within the surface layer, which in turn increases the extinction coefficient. The finer particles are most easily pressed into good contact, a higher extinction coefficient will result, and a more highly reflecting surface is obtained. This is in accordance with the facts here presented.

Similar effects on the reflecting power of the three powdered materials, namely, marble, precipitated calcium carbonate, and quartz, were observed when they were compressed. The intensity of the reflection from plates of the pressed powders was found to vary inversely as the particle size. Evidence has been brought forth in the case of precipitated calcium carbonate to show that if any powdered material can be obtained with a sufficiently small particle size, plates can be formed of them which will approach very closely the reflecting power of the natural crystal. Doubling the pressure increases the reflecting power by practically a constant amount until particles of 3 micron size are used.

[7] J. W. Mellor: *A Comprehensive Treatise on Inorganic and Theoretical Chemistry*, pp. 816–818.

[8] Int. Crit. Tab. 5, 259 (1929).

Pressure increments over 10,000 pounds per square inch rapidly become less and less important for particles 3 microns or less in size, the two curves gradually merging into one another. Probably for particles smaller than 1 micron, a pressure considerably less than 10,000 pounds per square inch would be sufficient to yield a plate with nearly as high a reflecting power as those here obtained.

It is known that the maximum of the reflected energy from plates prepared with pressed powders is at the same wave-length as the maximum reflected energy from a single crystal of the same material.[2] As reststrahlen plates are employed so frequently in infrared measurements, it is hoped that the material presented in this paper may be of value for those wishing to prepare them out of substances which are not available in large single crystals.

In conclusion, the author wishes to express his indebtedness to Professor R. W. Wood who suggested the problem and who was a constant source of advice during the course of the research. The author also wishes to thank Professors K. F. Herzfeld and A. H. Pfund for critically reading the manuscript.

Erratum

The Development of the Photographic Objective

R. KINGSLAKE, *Institute of Applied Optics, University of Rochester, Rochester, N. Y.*

(J. O. S. A. **42**, 73, 1934)

In the paper of the above title, page 84, column 1, line 24, the words "dotted" and "full" should be interchanged.